The Equations of Dialysis

NANT – New York - 2012

John A. Sweeny

jsweeny@tampabay.rr.com

An Equation is…

- **Math:**
  - "A statement that each of two statements is equal to each other."

$$Y^2 = 3x^3 + 2x + 7$$

- **Chemistry:**
  - " A symbolic expression that represents a chemical change as observed in a laboratory".

$$2H_2O = H_3O^+ + OH^-$$

- **Medical:**
  - " An expression made up of two members connected by the sign of equality".

Clearance x Time = Volume (Kt/V = 1)

Equation Types

- Hypothesis
  - Relationships implied without supporting evidence
- Empirical
  - Based solely on experiment and observation
  - No reference to scientific principals
- Theoretical
  - A formulation of apparent relationships
  - Deals with science concepts and knowledge
  - Implies considerable evidence of support
  - Pure science as opposed to applied science

Equation Relationships

- **The Equation:**

$$W_F = \frac{C - 2200}{E} \; x \, W_I$$

W_F = Final Weight

W_I = Initial Weight

C = Calorie intake/day

E = Exercise/day

2200 = Calories to maintain weight

- **The Relationships:**
  - **Final Weight is directly proportional to Calorie intake/day**

K/DOQI Guidelines for Classification

Stage	Description	GFR (mL/min)	Action
1	Damage with normal or high GFR	>90	CVD risk reduction; diagnose and treat; slow progression
2	Mild decrease in GFR	60-89	Monitor progression; nutritional assessment and intervention
3	Moderate decrease in GFR	30-59	Evaluate and treat complications
4	Severe decrease in GFR	15-29	Prepare for replacement therapy
5	Kidney Failure	<15	Replacement therapy if uremia is present

Question

Is there a way to determine which classification a patient falls into and what information do we need to know to figure this out?

MDRD Study Equation for calculating GFR

GFR (mL/min per 1.73 m^2 body surface area) =

$$186 \times (S_{Cr})^{-1.154} \times (Age)^{-0.203} \times (0.742 \text{ if female}) \times (1.210 \text{ if African-American})$$

S_{Cr} = serum creatinine measured in mg/dL

- **Not validated in:**
  - Diabetic kidney disease
  - Patients with serious comorbid conditions
  - Normal persons
  - Persons older than 70.

MDRD = Modification of Diet in Renal Disease

GFR from MDRD Study Equation

Results for a male (1.73 m^2)

S_{Cr} (Age = 60)	GFR	Age (SCR = 2)	GFR
1	81.0	40	39.5
2	36.4	50	37.8
3	22.8	60	36.4
4	16.4	70	36.3

Female = 26% lower

African America Male = 21% higher

African American Female = 10% lower

Question

If we are going to treat the patient, we need a way to measure our success. Urea is the major marker used.

Is there a way to know how much urea a patient will generate based on their diet intake of protein?

The Conversion Equation

Protein to Urea Nitrogen

$$DPI = PCR = 9.35\,G + 11.04$$

or

$$G = (PCR - 11.04) / 9.35$$

DPI = Dietary Protein Intake (grams/day)

PCR = Protein Catabolic Rate (grams/day)

G = Generation Rate (milligrams of urea nitrogen/minute)

BUN Increase in a ESRD Patient

- **Patient's Weight = 70.0 kilograms**
- **Patient's Fluid Volume = 58% of Weight**
- **70 x 0.58 = 40.6 liters = 406 deciliters**

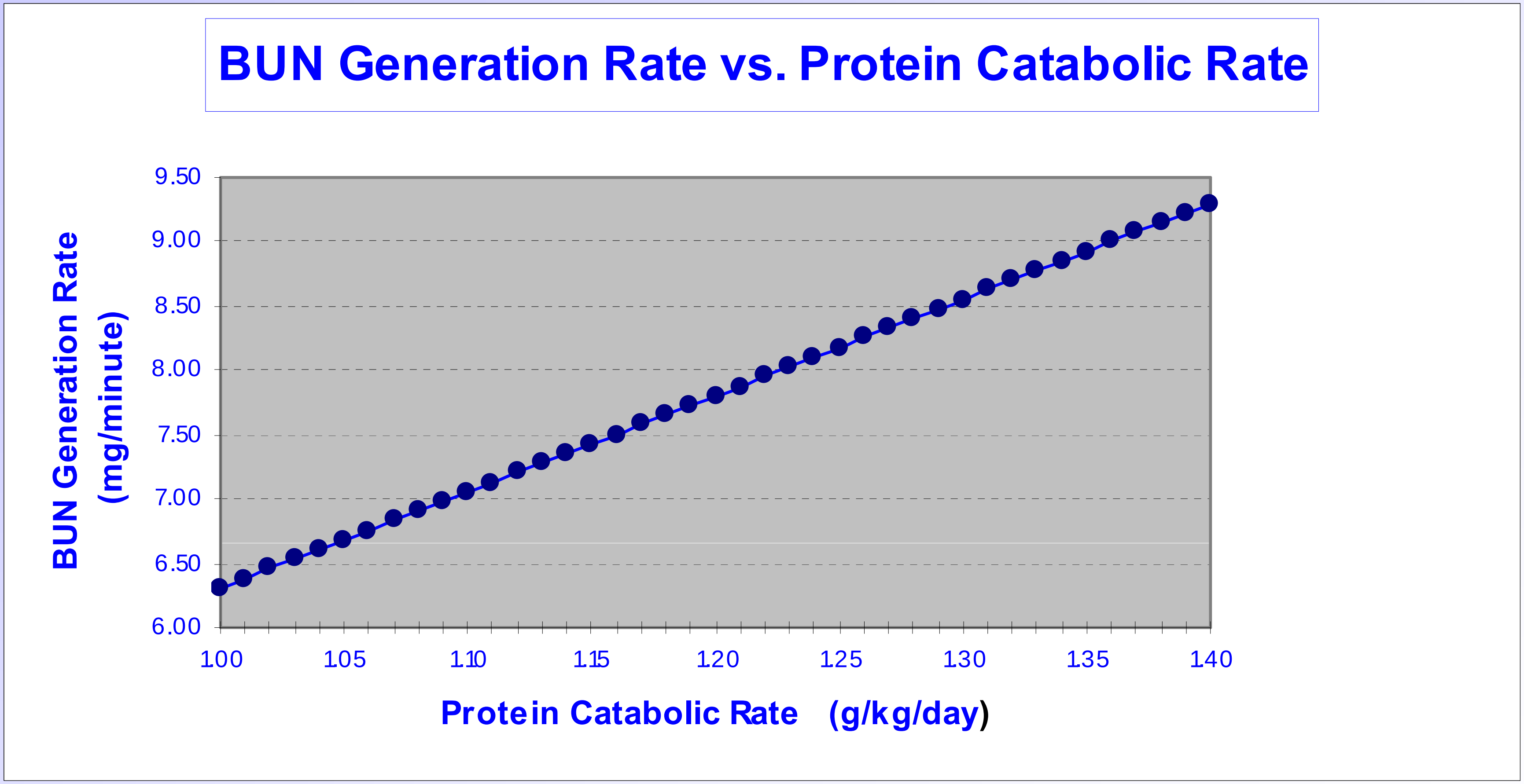
Let DPI = PCR = 84 g/day (1.2 g/kg/day), then:

G = (84 - 11.04) / 9.35 = 7.80 mg/min.

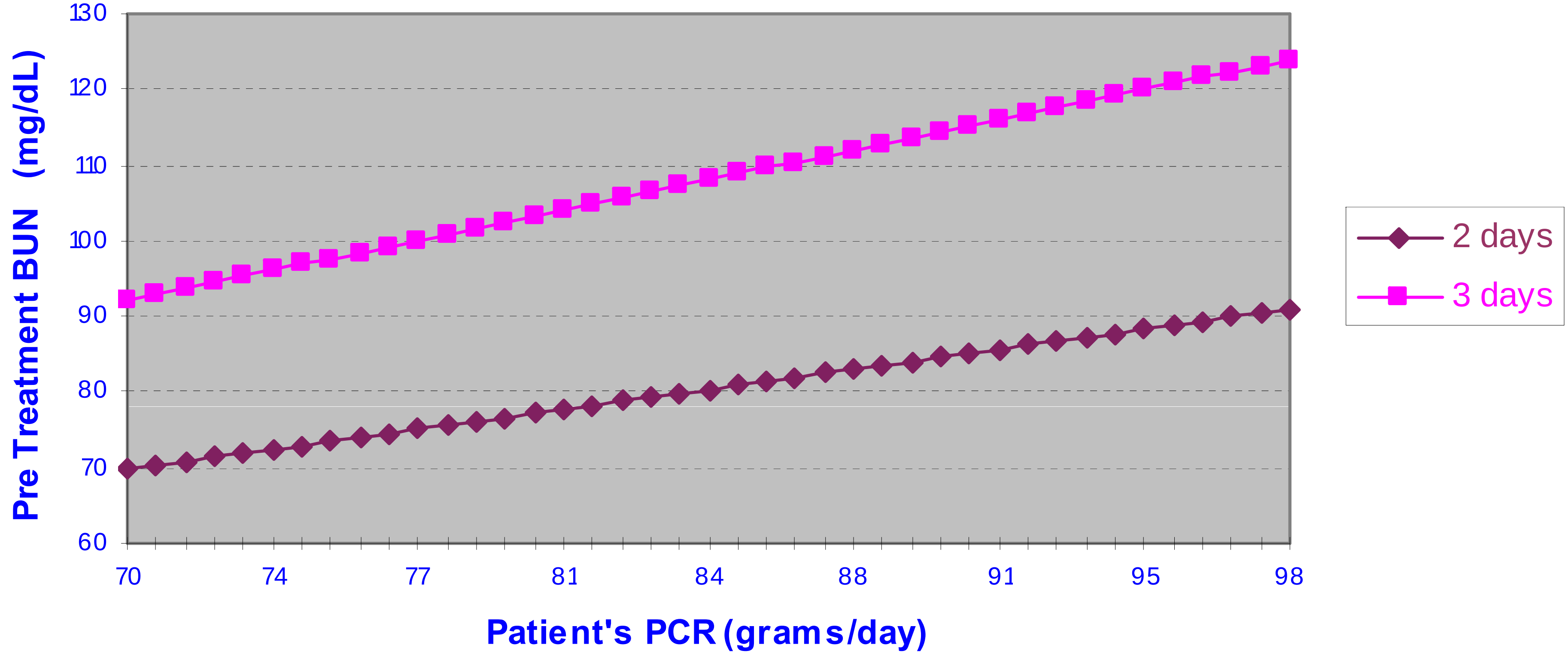
7.80 mg/min x 60 min/hr = 468 mg/hr.

468 mg/hr x 24 hr/day = 11,232 mg/day

11,232 mg/day / 406 dL = 27.7 mg/dL/day.


BUN Generation Rate vs. Protein Catabolic Rate
BUN Generation Rate
(mg/minute)
9.50
9.00
8.50
8.00
7.50
7.00
6.50
6.00
1.00
1.05
1.10
1.15
1.20
1.25
1.30
1.35
1.40
Protein Catabolic Rate (g/kg/day)

Patient's PCR vs. Pre Treatment BUN



Question

Since the patient will need to have his/her urea removed, and there are so many different dialyzers, is there a simple way to measure urea removal performance for a given dialyzer?

Dialyzer BUN Clearance

THE EMPIRICAL FORMULA FOR BLOOD CLEARANCE IS:

$$C_{BUN} = \left(\frac{A_{BUN} - V_{BUN}}{A_{BUN}} \right) Q_B$$

WHERE:

C_{BUN} = CLEARANCE OF SOLUTE X. (mL/min)

A_{BUN} = ARTERIAL CONCENTRATION OF X. (mg/dL)

V_{BUN} = VENOUS CONCENTRATION OF X. (mg/dL)

Q_B = BLOOD FLOWRATE (mL/min)

BUN of Venous Blood based on Dialyzer KoA

QB = 300 mL/min QD = 600 mL/min

KoA (mL/min)	Clearance (mL/min)	Arterial Blood BUN Values (mg/dL) 70	80	90	100	110
300	169	30.6	34.9	39.3	43.7	48.0
400	196	24.3	27.7	31.2	34.7	38.1
500	217	19.4	22.1	24.9	27.7	30.4
600	232	15.9	18.1	20.4	22.7	24.9
700	245	12.8	14.7	16.5	18.3	20.2
800	254	10.7	12.3	13.8	15.3	16.9
900	262	8.9	10.1	11.4	12.7	13.9
1000	269	7.2	8.3	9.3	10.3	11.4

Question

It's not very practical to measure blood urea concentrations to determine clearance.

Is there an equation that can calculate the expected clearance based on a known blood flowrate, dialysate flowrate and dialyzer used?

Determining the K_OA for a Dialyzer

$$KoA = \left[Q_B \Big/ 1 - \frac{Q_B}{Q_D} \right] 1n \left[\frac{1 - \frac{C_X}{Q_D}}{1 - \frac{C_X}{Q_B}} \right]$$

Where: **C_X = Clearance of solute, X**

Q_B = Blood flowrate

Q_D = Dialysate flowrate

ln = Natural logarithm

= e = 2.718281828......

KoA Calculation

Enter known values for Clearance, Blood Flowrate, and Dialysate Flowrate.
Note: The formula will not work if the Blood and Dialysate Flowrates are equal.
For this condition, substitute a Blood or Dialysate Flowrate which is changed by only one mL. The error will be less than one in the clearance value.

Dialyzer	Clearance	Blood Flowrate	Dialysate Flowrate
IP4U	225	300	600

KoA Formula =

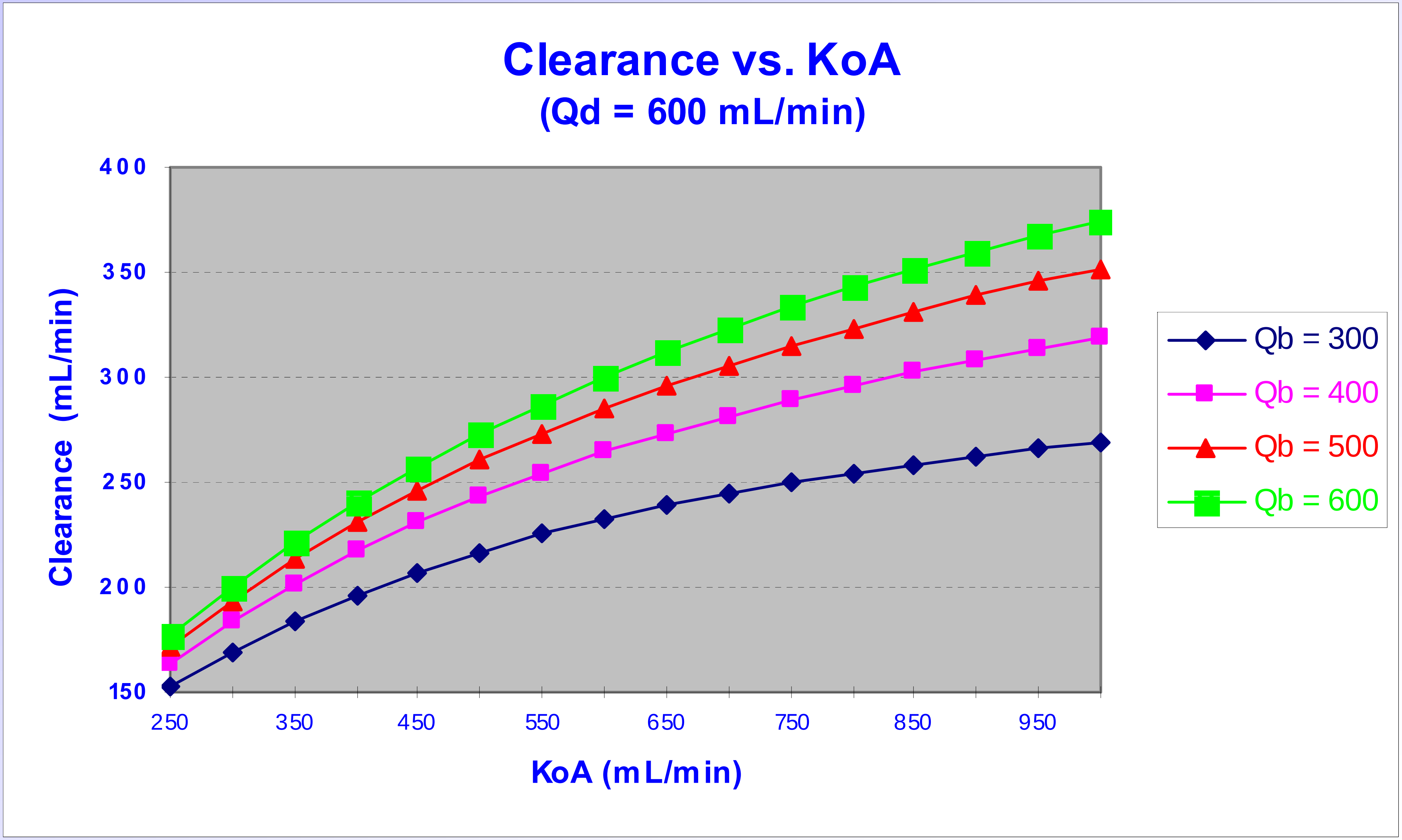
$$KoA = \left[Q_B \Big/ 1 - \frac{Q_B}{Q_D} \right] 1n \left[\frac{1 - \frac{C_X}{Q_D}}{1 - \frac{C_X}{Q_B}} \right]$$

KoA = 550 mL/min.

Calculating BUN Clearance using KoA

$$C_{BUN} = \frac{Q_B\left(e^{KoA\left(\frac{1}{Q_B}-\frac{1}{Q_D}\right)}-1\right)}{e^{KoA\left(\frac{1}{Q_B}-\frac{1}{Q_D}\right)}-\frac{Q_B}{Q_D}}$$

C_x = Clearance of x.
Q_B = Blood Flowrate
Q_D = Dialysate Flowrate
KoA = Clearance Coefficient
e = 2.718281828….



Clearance vs. KoA
(Qd = 600 mL/min)
400
350
300
250
200
150
Clearance (mL/min)
250
350
450
550
650
750
850
950
KoA (mL/min)
Qb = 300
Qb = 400
Qb = 500
Qb = 600

Clearance for a Dialyzer with a KoA = 600 mL/min

BLOOD FLOW ml/min	DIALYSATE FLOW ml/min				
	400	500	600	700	800
200	175	179	181	183	184
300	217	226	232	237	240
400	240	255	264	271	276
500	255	273	285	294	301

Question

Once the fluid volume of the patient is known and the dialyzer clearance calculated, is there an equation to determine the time of dialysis?

Length of Treatment

$Kt/V = 1.3$

K = Dialyzer Clearance (mL/min)

t = Time of Treatment (min)

V = Patient's volume (mL)

$t = (1.3 \times V)/K$

Calculating Kt/V

Dialyzer Clearance (K) (mL/min)	Treatment Time (t) (minutes)	Patient Weight (kilograms)
250	240	70

Patient Volume (V) (milliliters)	Kt/V
40600	1.48

Time vs. Kt/V

Kt/V	Tx Time (minutes)
0.90	146
1.00	162
1.10	179
1.20	195
1.30	211
1.40	227
1.50	244
1.60	260

Question

Once the clearance of the dialyzer and time of treatment are known, is there a way to estimate how the urea is reduced in the patient while she/he is being dialyzed?

Urea Reduction Equation

$$C = C_0\, e^{-Kt/V} + G/K\,(1 - e^{-Kt/V})$$

C = Plasma BUN Concentration (mg/mL)*

C_0 = Predialysis BUN Concentration (mg/mL)*

K = Dialyzer Clearance (mL/min)

t = time (minutes)

V = Patient Volume (mL)

G = Generation of urea (mg/min)

*** mg/mL equals mg/dL divided by 100.**

$$C = C_0 e^{-Kt/V} + G/K (1 - e^{-Kt/V})$$

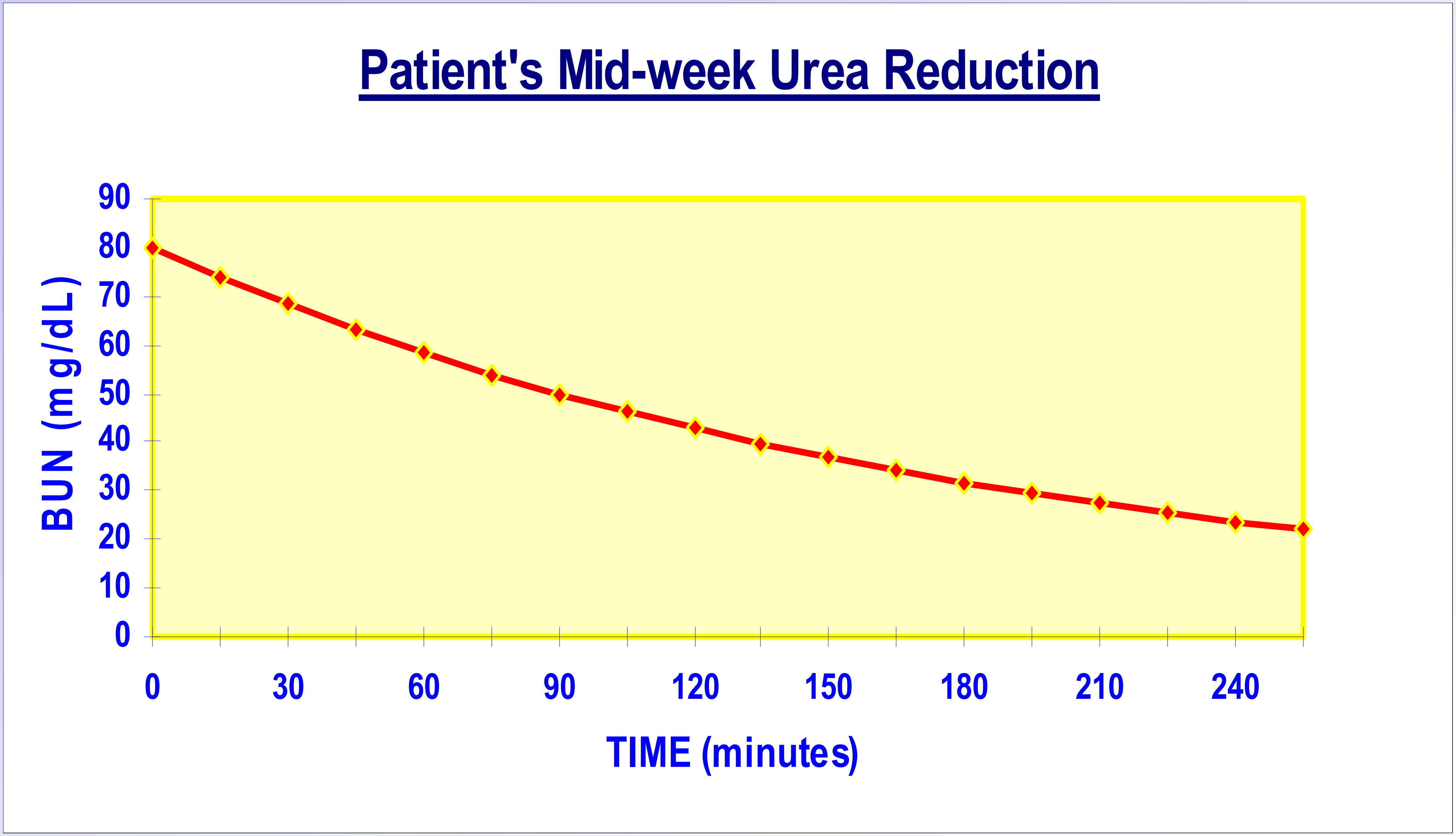
C = Plasma BUN Concentration (mg/mL)*
C_0 = Predialysis BUN Concentration (mg/mL)*
K = Dialyzer Clearance (mL/min)
t = time (minutes)
V = Patient Volume (mL)
G = Generation of urea (mg/min)

* mg/mL equals mg/dL divided by 100.

C_0 =	80	mg/dL	K =	225	mL/min
G =	8.5	mg/min	V =	40600	mL
t =	240	minutes	C =	23.9	mg/dL

$C = C_0(e^{-Kt/V}) + G/K(1 - e^{-Kt/V})$

Time	C	$C_0(e^{-Kt/V})$	$G/K(1 - e^{-Kt/V})$
(minutes)	(mg/dL)	(mg/dL)	(mg/dL)
0	80.0	80.0	0.0
60	58.5	57.4	1.1
120	42.9	41.1	1.8
180	31.9	29.5	2.4
240	24.0	21.2	2.8


Patient's Mid-week Urea Reduction
BUN (mg/dL)
90
80
70
60
50
40
30
20
10
0
0
30
60
90
120
150
180
210
240
TIME (minutes)

Question

The higher the reduction in urea, the better the treatment, but we can't run the patient all day. Is there a urea reduction value that defines an adequate treatment?

$$URR\ \% = (C_{PRE} - C_{POST}) / C_{PRE} = 1 - (C_{POST}/C_{PRE})$$

C_{PRE} TREATMENT (mg/dL)	C_{POST} TREATMENT (mg/dL)											
	50	48	45	42	40	38	35	32	30	28	25	22
140	64%	66%	68%	70%	71%	73%	75%	77%	79%	80%	82%	84%
135	63%	64%	67%	69%	70%	72%	74%	76%	78%	79%	81%	84%
130	62%	63%	65%	68%	69%	71%	73%	75%	77%	78%	81%	83%
125	60%	62%	64%	66%	68%	70%	72%	74%	76%	78%	80%	82%
120	58%	60%	63%	65%	67%	68%	71%	73%	75%	77%	79%	82%
115	57%	58%	61%	63%	65%	67%	70%	72%	74%	76%	78%	81%
110	55%	56%	59%	62%	64%	65%	68%	71%	73%	75%	77%	80%
105	52%	54%	57%	60%	62%	64%	67%	70%	71%	73%	76%	79%
100	50%	52%	55%	58%	60%	62%	65%	68%	70%	72%	75%	78%
95	47%	49%	53%	56%	58%	60%	63%	66%	68%	71%	74%	77%
90	44%	47%	50%	53%	56%	58%	61%	64%	67%	69%	72%	76%
85	41%	44%	47%	51%	53%	55%	59%	62%	65%	67%	71%	74%
80	38%	40%	44%	48%	50%	53%	56%	60%	63%	65%	69%	73%
75	33%	36%	40%	44%	47%	49%	53%	57%	60%	63%	67%	71%
70	29%	31%	36%	40%	43%	46%	50%	54%	57%	60%	64%	69%
65	23%	26%	31%	35%	38%	42%	46%	51%	54%	57%	62%	66%
60	17%	20%	25%	30%	33%	37%	42%	47%	50%	53%	58%	63%
55	9%	13%	18%	24%	27%	31%	36%	42%	45%	49%	55%	60%

Question

If the urea concentration at the beginning and end of the treatment are known, is there a relationship between Kt/V and these values?

Urea Reduction Ratio vs. Kt/V

$$URR = (1 - R)$$

$$R = C_{post}/C_{pre}$$

C_{pre} = Pre Treatment Plasma urea level

C_{post} = Post Treatment Plasma urea level

$$Kt/V = -\ln(R - 0.03) + (4 - 3.5R) \times UF/W$$

UF = Fluid removed (liters)

W = Postdialysis weight (kilograms)

Treatment Kt/V

W = 70 kg, R =0.30, URR = 70%

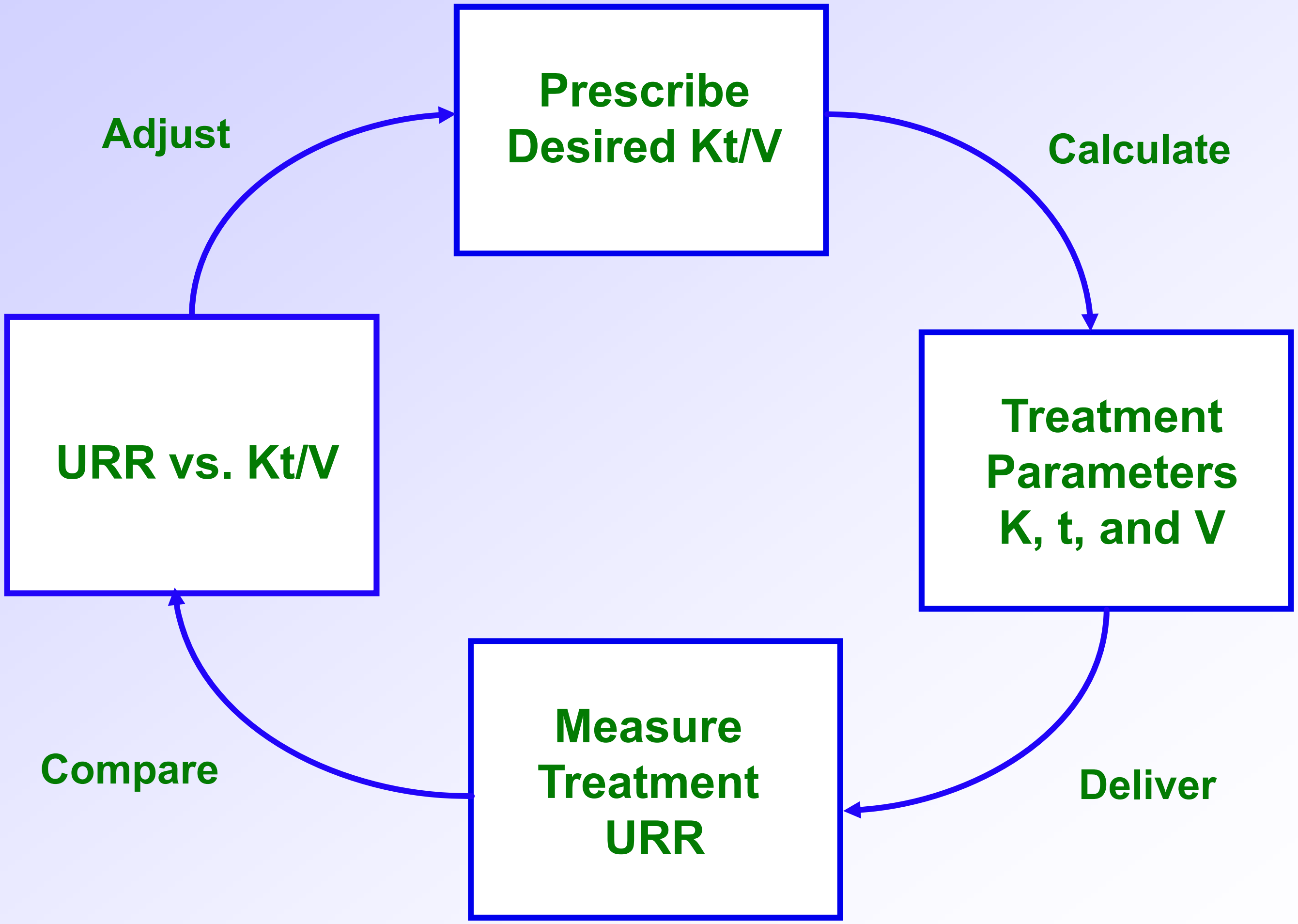
UF (Liters)	Equation 1	Equation 2	Equation 3
0	1.31	1.31	1.31
1	1.35	1.35	1.36
2	1.39	1.39	1.40
3	1.44	1.44	1.45
4	1.48	1.48	1.50
10	1.73	1.81	1.78

Equation 1 = -ln(R – 0.03) + (4 – 3.5R) x UF/W

Equation 2 = -ln(R – 0.03 – (0.75 x UF/W))

Equation 3 = 2.2 – 3.3 x (R – 0.03 – UF/W)

Achieving Quality Therapy



Handy Technical Facts

- Q_B actual = 9% less than equipment display at negative 200 mmHg pre-pump pressure.
- For water: Parts per million (ppm) x 1.5 = Conductivity (µS/cm)
- Dialysate conductivity (mS/cm) =

  $(Na^+ + K^+ + Mg^{++} + Ca^{++} - 6)$ mEq/L / 10
- Blood pH = 6.1 + log ($[HCO_3^-]$ / 0.03 x Pco_2 mmHg)

  Normal $[HCO_3^-]$ = 24 mEq/L   Normal PCO2 = 40 mmHg
- Hardness (mg $CaCO_3$/L) = 2.497 x Ca^{++} (mg/L) + 4.118 x Mg^{++} (mg/L)
- 1 atmosphere = 760 mmHg = 14.7 PSI = 101.325 kilopascals
- 1 mOsm = 19.3 mmHg @ 37^O C

References

- Handbook of Dialysis – John T. Daugirdas, Todd S. Ing, Peter G. Blake, 4th Edition, © 2007, Lippincott, Williams and Wilkens., ISBN 13: 978-0-7817-5253-4
- NKF Practice Guidelines for Chronic Kidney Disease: Evaluation, Classification, and Stratification – Andrew S. Levey, MD, et al, Annals of Internal Medicine, Volume 139, No. 2, July 2003
- Water Treatment for Dialysis – Douglas A. Luehmann, et al, NANT Publication, July 1989
- Review of Hemodialysis for Nurses and Dialysis Personnel – Judith Z. Kallenbach, 8th Edition, 2012, Mosby Inc., ISBN 978-0-323-07702-6